2025

(Nov–Dec)

FYUP : 5th Semester Examination

MINOR

MATHEMATICS

(Elementary Algebra)

MTH–302

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **four** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Define subset of a set. Show that if

$$A = \{x : x \in R,\ x^2 - 4x + 10 > 0\}$$
$$B = \{1,\ 2,\ 3,\ 4\}$$

then $B \subset A$. 1+4=5

(b) Show that the relation R on $\mathbb{R}$ defined as

$$R = \{(a, b) : a \leq b^2\}$$

is neither reflexive nor symmetric nor transitive. 4

(c) Define exponential function. Show that e^x is one-one and onto on the interval $(0, \infty)$. 1+4=5

(d) Let $f : R \to R$ be a function defined by $f(x) = [x]$. Justify if f is one-one. Sketch roughly its graph on the interval $[2, 10]$. 4

2. (a) Show that for any sets A, B, $(A \cup B)' = A' \cap B'$ where A' stands for the complement of A. 4

(b) Examine whether the relation R on the set N of natural numbers given by $R = \{(x, y) : 3x - y = 0\}$ is reflexive, symmetric or transitive. 5

(c) Show that the function $f : A \to B$ is invertible if and only if f is one-one and onto. 5

(d) Let

$$f(x) = \frac{4x+3}{6x-4}, \quad g(x) = 2x, \quad x \neq \frac{2}{3}$$

Find $(f \circ f)(x)$, $(g \circ f)(x)$, $(g \circ g)(x)$. 4

UNIT—II

3. (a) Define transpose of a matrix. For

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 2 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 & 1 \\ 3 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

verify that $(A + B)' = A' + B'$ where A' = transpose of A. 5

(b) What do you mean by the inverse of a square matrix A? Verify that

$$B = \frac{1}{2}\begin{bmatrix} 0 & -1 & 2 \\ -4 & -1 & 8 \\ 2 & 1 & -4 \end{bmatrix}$$

is the inverse of

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix}$$ 5

(c) Give one example of a Hermitian matrix with justification. 4

(d) Show that the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

is nilpotent of index 3. 5

4. *(a)* Let A, B be two scalar matrices of order 3×3 over R. Verify that $AB = BA$. 5

(b) Define inverse of a square matrix. Verify that the matrices

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 4 & 2 \end{bmatrix} \text{ and } B = \frac{1}{20}\begin{bmatrix} 2 & 8 & -2 \\ -4 & 4 & 4 \\ 7 & -12 & 3 \end{bmatrix}$$

are inverses to each other. 5

(c) Give one example each of symmetric and skew-symmetric matrix with justification. 5

(d) Show that the matrix

$$A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

is idempotent. 4

UNIT—III

5. *(a)* Find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

by elementary operations. 5

(b) For

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 0 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

verify that $\text{Tr}(AB) = \text{Tr}(BA)$. 4

(c) Using determinant method, determine the rank of

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 4 & 2 \\ 6 & 2 & -2 \end{bmatrix}$$

5

(d) Solve the equations

$$x + y + 2z = 9, \ x + 2y + z = 8, \ 2x + 2y - z = 3$$

by matrix method. 5

6. *(a)* Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -1 \\ 1 & 0 & 3 \end{bmatrix}$$

Find A^{-1} (the inverse of A) using adjoint. 6

(b) Determine the rank of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{bmatrix}$$

by elementary operations. 4

(c) Given that

$$\begin{bmatrix} 3 & 0 & -3 \\ -2 & -4 & 4 \\ -5 & 2 & 1 \end{bmatrix}$$

is the adjoint of

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ 4 & 1 & 2 \end{bmatrix}$$

Solve the equations

$$2x + y + 2z = 6,\ 3x + 2y + z = 8,$$
$$4x + y + 2z = 8$$ 5

(d) Define the following : 4

(i) Row and column vectors of a matrix

(ii) Trace of a square matrix

(iii) Elementary matrix

UNIT—IV

7. (a) Let '$*$' be a binary operation defined on $\mathbb{Q}$ as $a * b = \frac{ab}{2}$. Determine if '$*$' is associative and commutative. 3

(b) Let G be a group and $a, b, c \in G$. Show that

$$a * b = a * c \Rightarrow b = c$$
and $$a * c = b * c \Rightarrow a = b$$ 3

(c) Show that if G is a group, then any element of G has unique inverse in G. 4

(d) Let n be a positive integer and $G = \{x \in \mathbb{C} : x^n = 1\}$. Show that G is a group under usual multiplication. 5

(e) Define order of a group and order of an an element. Give examples of a group G of order 4 and an element $a \in G$ of order 2. 4

8. (a) What do you mean by a group? Show that the set $\{1, \omega, \omega^2\}$ (where $\omega^3 = 1$) forms a group under usual multiplication. 2+3=5

(b) Let G be a group and $a \in G$ with order $(a) = n$. Show that $O(a^{-1}) = n$. 4

(c) Let G be a group and $a, b \in G$. Show that $(ab)^{-1} = b^{-1}a^{-1}$. 3

(d) Let G be a group and $a, b \in G$. Show that $ax = b$ and $ya = b$ have unique solutions in G. 4

(e) Consider the group $G = \{e, a, b, c\}$ with the binary operation $*$ defined as $e * x = x * e = x,\ x * x = e\ \ \forall\ x \in G$ and

$$a * b = b * a = c,\ a * c = c * a = b,$$
$$b * c = c * b = a$$

Construct the group table for G. 3

★ ★ ★